

# Warm up

Expand and simplify the following expressions:

1	$2(a + b) + b =$	?
2	$\frac{1}{2}(a - b) + b =$	?
3	$a + \frac{1}{2}(a + b) =$	?
4	$a - 2(a - 2b) =$	?
5	$\frac{1}{3}(a + 2b) - b =$	?

# Vectors Quiz!

Mini whiteboards  
ready?

## Vectors Quiz!

What is  $\overrightarrow{PQ} + \overrightarrow{QR}$ ?

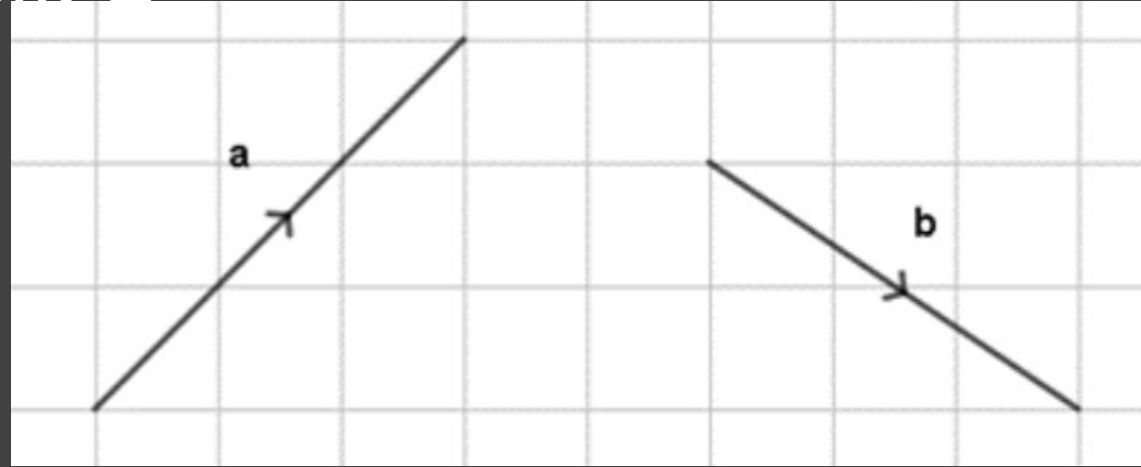
a.  $\overrightarrow{RQ}$

b.  $\overrightarrow{PR}$

c.  $\overrightarrow{QP}$

d.  $\overrightarrow{RP}$

2. This diagram shows vectors **a** and **b**



a.  $a + b$

b.  $a + 2b$

c.  $a - b$

d.  $b - a$

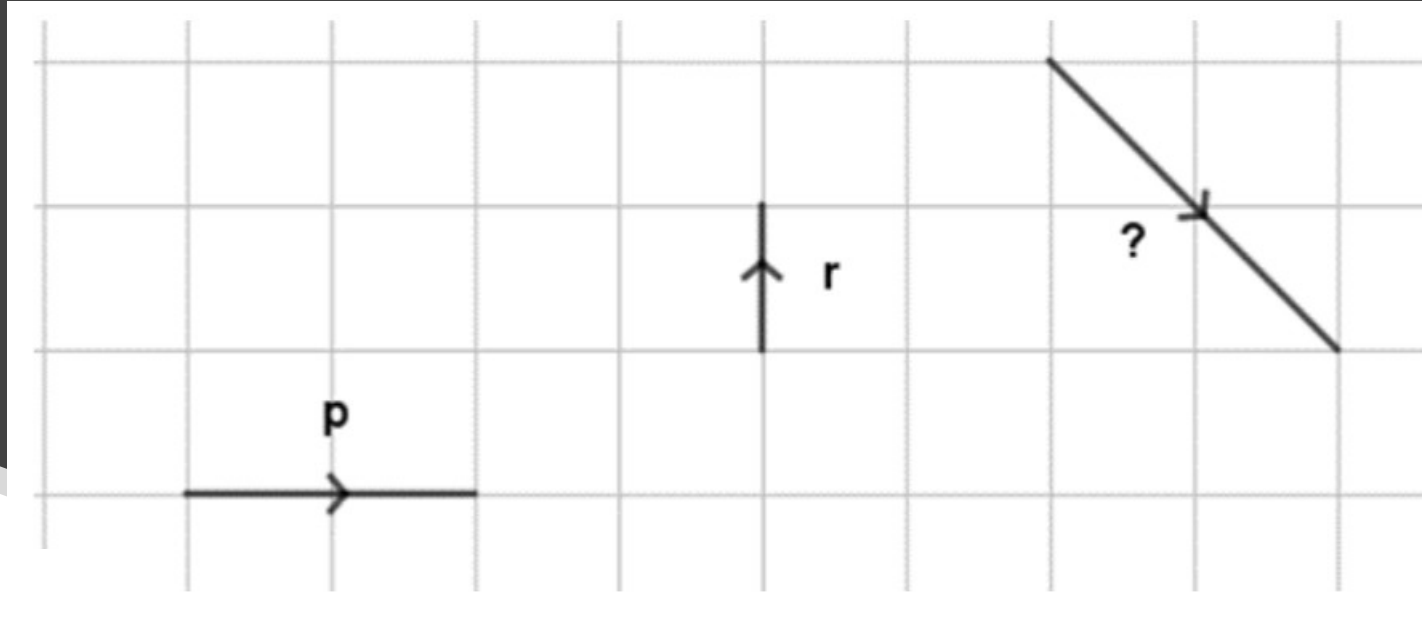
Which  
vector is  
shown in  
this  
diagram to  
the right?



## Vectors Quiz!

- a.  $\mathbf{p} - \mathbf{q}$
- b.  $2\mathbf{a}$
- c.  $-\mathbf{a}$
- d.  $\mathbf{q} - \mathbf{p}$

If  $\overrightarrow{PQ} = \mathbf{a}$ , what is  $\overrightarrow{QP}$ ?



a.  $p - r$

b.  $p - 2r$

c.  $p + 2r$

d.  $p + r$

4. What is the missing vector?

# Vectors Quiz!

a.  $\begin{pmatrix} 10 \\ 3 \end{pmatrix}$

b.  $\begin{pmatrix} 24 \\ -18 \end{pmatrix}$

c.  $\begin{pmatrix} 64 \\ -36 \end{pmatrix}$

d.  $\begin{pmatrix} 10 \\ 9 \end{pmatrix}$

5. Complete  
this vector  
addition

$$\begin{pmatrix} 6 \\ -3 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \end{pmatrix} =$$

## Vectors Quiz!

- 1) They are parallel
- 2) AB is  $k$  times the length of BC.

If  $\overrightarrow{AB} = k\overrightarrow{BC}$ , what do we know about lines AB and BC?

Write down two things.



# Vectors Quiz!

a.  $\begin{pmatrix} -2 \\ -2 \end{pmatrix}$

b.  $\begin{pmatrix} 2 \\ -12 \end{pmatrix}$

c.  $\begin{pmatrix} -3 \\ 2 \\ 7 \\ 5 \end{pmatrix}$

d.  $\begin{pmatrix} 10 \\ -2 \end{pmatrix}$

7. Complete this vector subtraction

$$\begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} -6 \\ 7 \end{pmatrix} =$$

# Vectors Quiz!

a.  $\begin{pmatrix} 10 \\ -4 \end{pmatrix}$

b.  $\begin{pmatrix} 4 \\ -7 \end{pmatrix}$

c.  $\begin{pmatrix} 4 \\ 13 \end{pmatrix}$

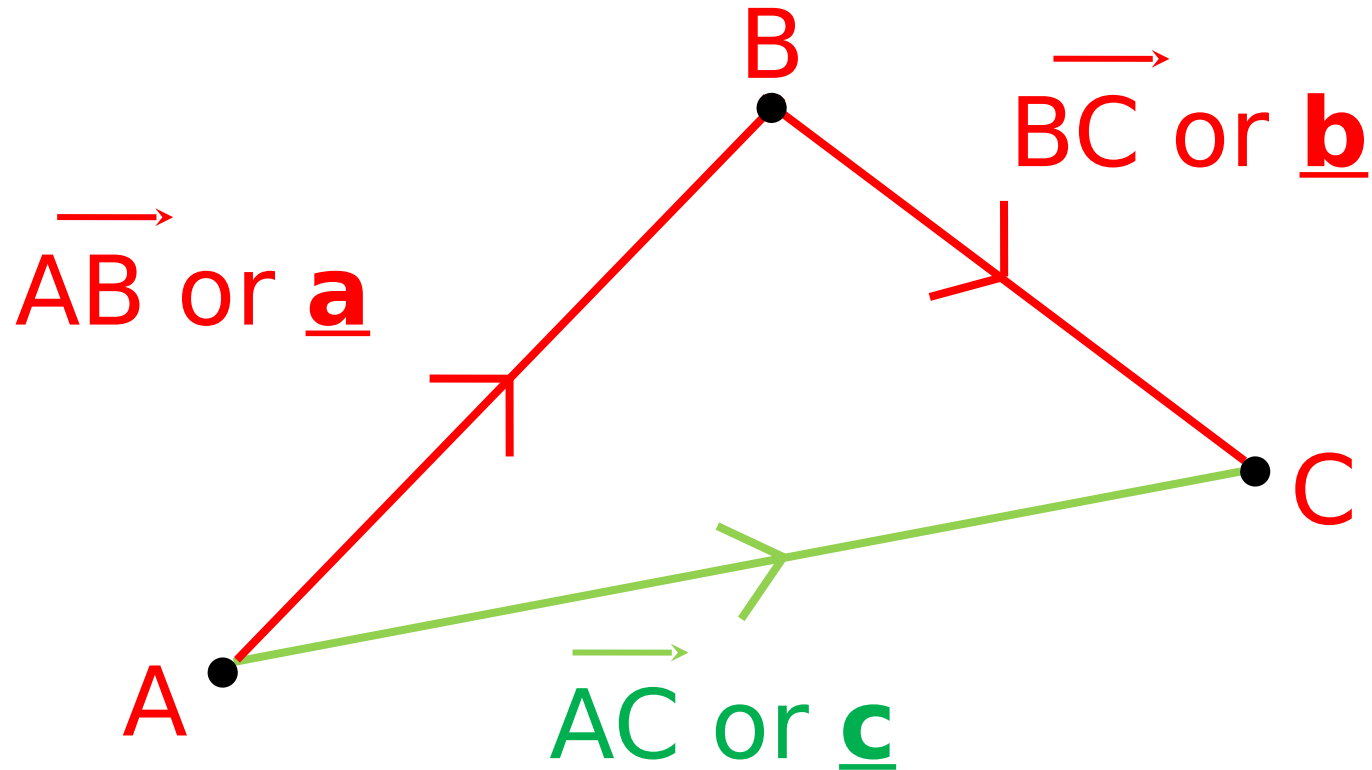
d.  $\begin{pmatrix} 25 \\ -22 \end{pmatrix}$

5.  $\mathbf{g} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}, \mathbf{h} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

Work out the vector  $2\mathbf{g} + \mathbf{h}$ .

None of  
them!  $\begin{pmatrix} 4 \\ -13 \end{pmatrix}$

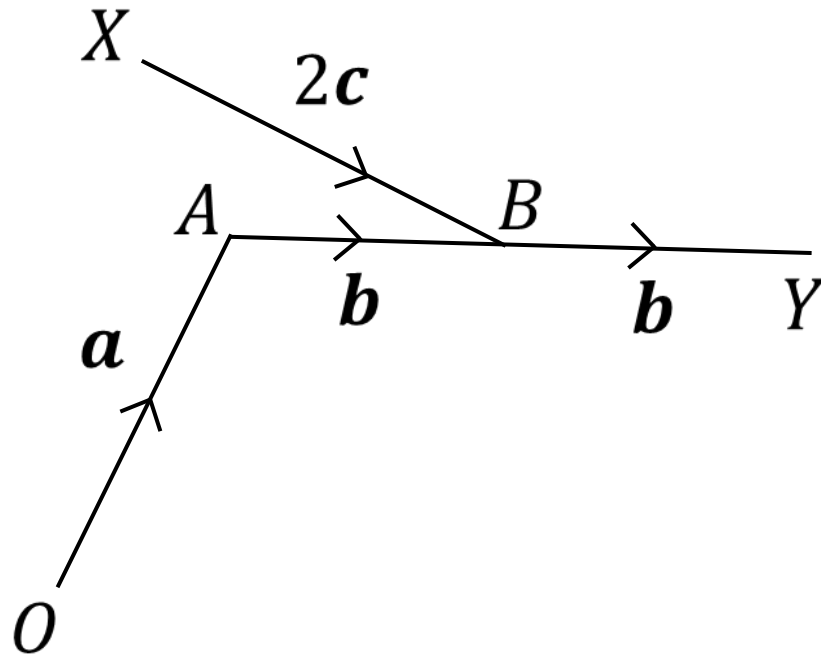
# Resultant Vectors



A resultant vector is a single vector which is equivalent to a set of vectors

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} \quad \text{or} \quad \underline{c} = \underline{a} + \underline{b}$$

# Adding and Subtracting Vectors

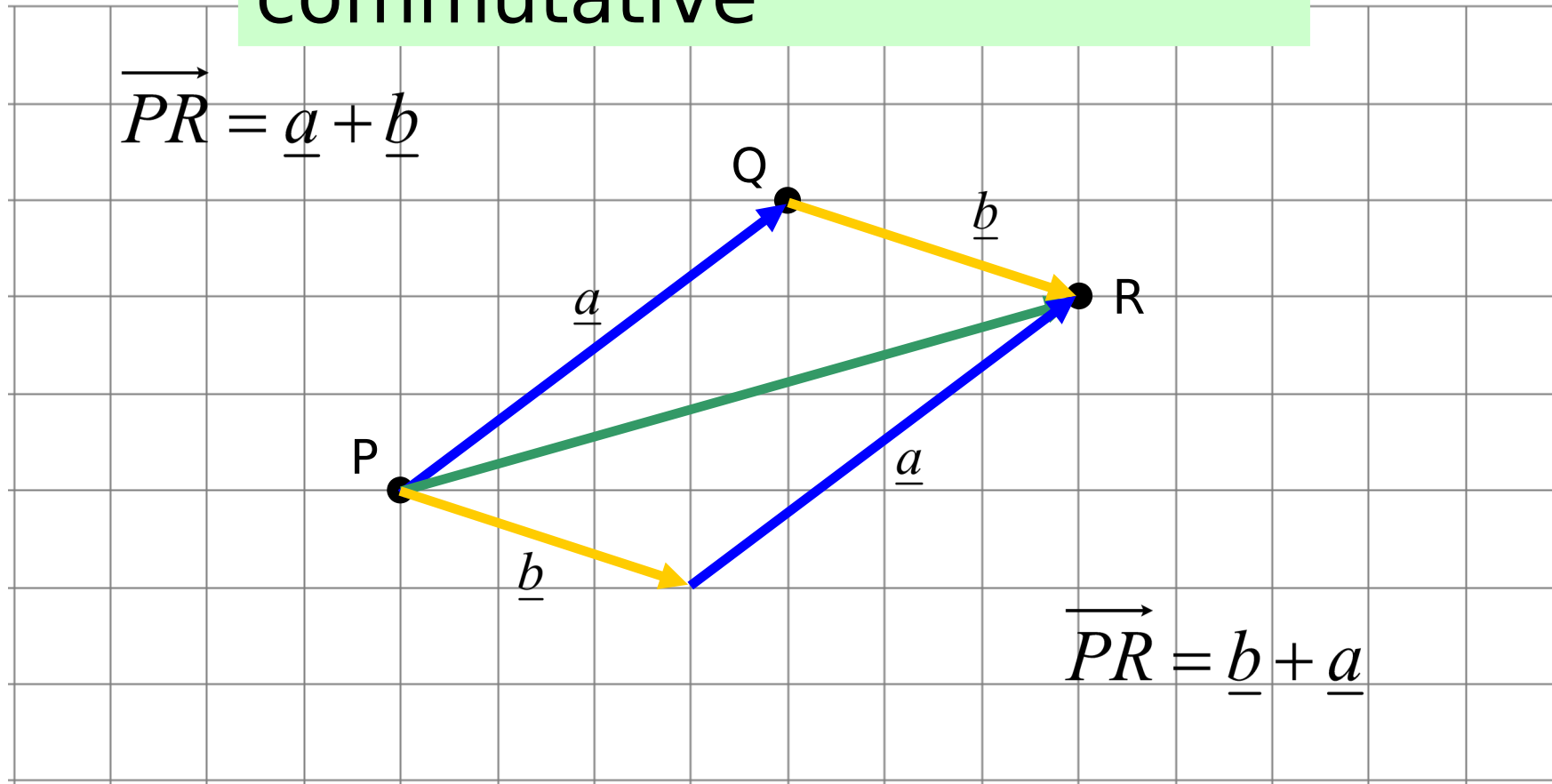


If  $\vec{OA} = a$ ,  $\vec{AB} = b$  and  $\vec{XB} = 2c$ ,  
then find the following in terms  
of  $a$ ,  $b$  and  $c$ :

$\vec{OB} =$	<input data-bbox="1671 529 1959 639" type="text" value="?"/>
$\vec{OY} =$	<input data-bbox="1671 644 1959 753" type="text" value="?"/>
$\vec{AX} =$	<input data-bbox="1671 758 1982 839" type="text" value="?"/>
$\vec{XO} =$	<input data-bbox="1671 843 2046 939" type="text" value="?"/>
$\vec{YX} =$	<input data-bbox="1671 943 2046 1039" type="text" value="?"/>

**Note:** to go in the opposite direction we need to subtract!

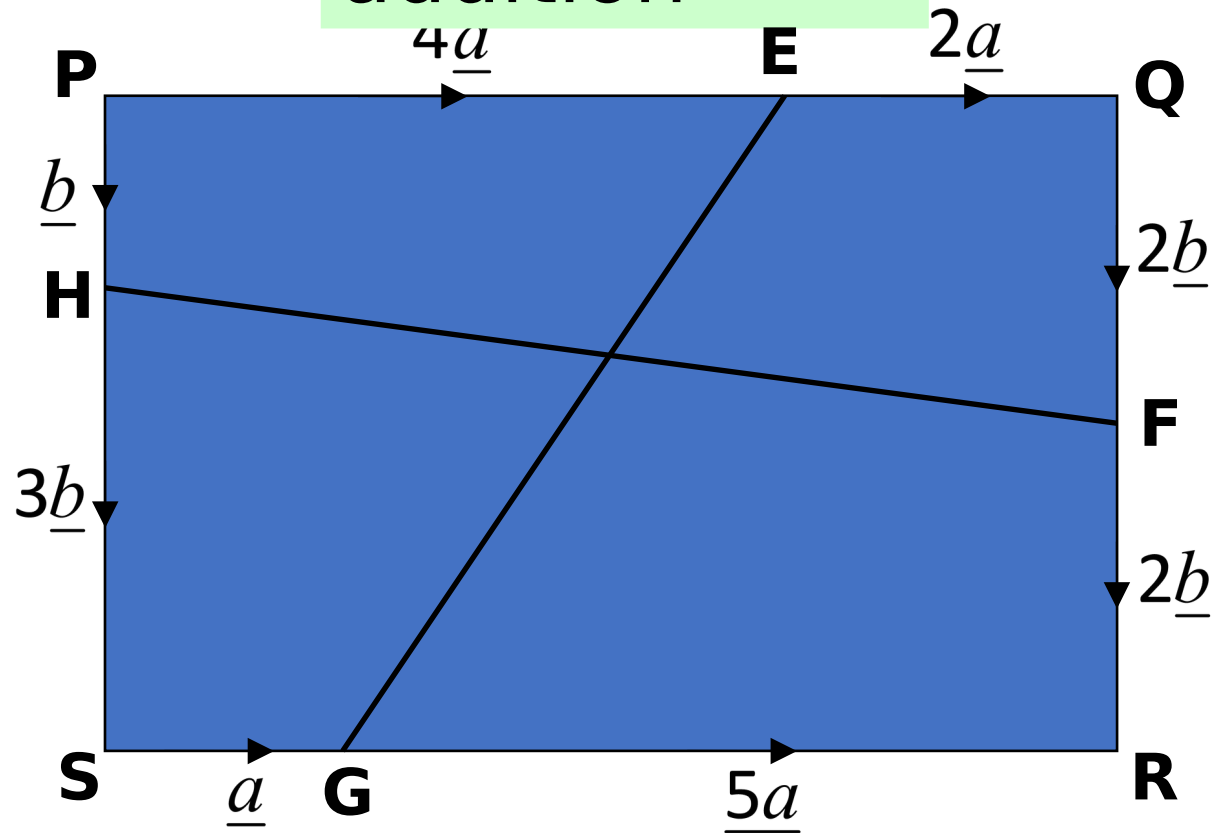
# Vector addition is commutative



Like normal addition, order doesn't matter when adding vectors

$$\underline{a} + \underline{b} = \underline{b} + \underline{a}$$

# Vector addition



On your mini white boards, write down vectors, in terms of  $\underline{a}$  and  $\underline{b}$  for:

$\overrightarrow{EF}$   $\overrightarrow{PR}$   $\overrightarrow{QP}$   $\overrightarrow{EG}$   $\overrightarrow{GE}$   $\overrightarrow{RH}$   $\overrightarrow{HR}$   $\overrightarrow{SQ}$   $\overrightarrow{FH}$

# Vector Geometry: problem solving

Geometric problems can be solved using the rules for adding and subtracting **vectors** and multiplying

EXAMPLE **vectors by a scalar.**

1 OABC is a parallelogram.

M is the midpoint of AB.

$OA = \mathbf{a}$  and  $OC = \mathbf{c}$

Find in terms of  $\mathbf{a}$  and  $\mathbf{c}$ .

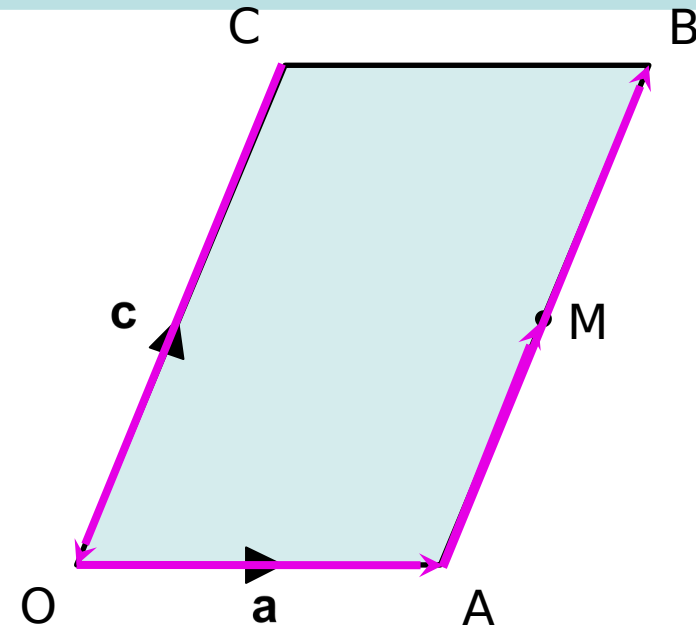
**a**  $CB = OA = \mathbf{a}$

**b**  $BA = -OC = -\mathbf{c}$

**c**  $OB = OA + AB = \mathbf{a} + \mathbf{c}$

**d**  $CA = CO + OA = -\mathbf{c} + \mathbf{a}$

**e**  $OM = OA + AM = OA + \frac{1}{2}AB =$



# Vector Geometry

Geometric problems can be solved using the rules for adding and subtracting **vectors** and multiplying vectors by a **scalar**.

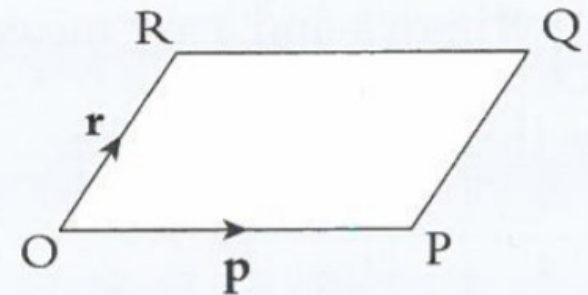
Q  
1

QR is a parallelogram.

$\vec{OP} = \mathbf{p}$  and  $\vec{OR} = \mathbf{r}$ .

Find the following in terms of  $\mathbf{p}$  and  $\mathbf{r}$

a  $\vec{RQ}$ ,    b  $\vec{QP}$ ,    c  $\vec{OQ}$ ,    d  $\vec{PR}$ .





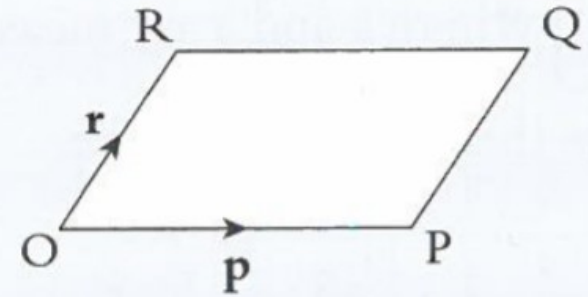
# Vector Geometry

OPQR is a parallelogram.

$\vec{OP} = \mathbf{p}$  and  $\vec{OR} = \mathbf{r}$ .

Find the following in terms of  $\mathbf{p}$  and  $\mathbf{r}$

a  $\vec{RQ}$ ,    b  $\vec{QP}$ ,    c  $\vec{OQ}$ ,    d  $\vec{PR}$ .



a  $\vec{RQ} = \mathbf{p}$

b  $\vec{QP} = -\mathbf{r}$

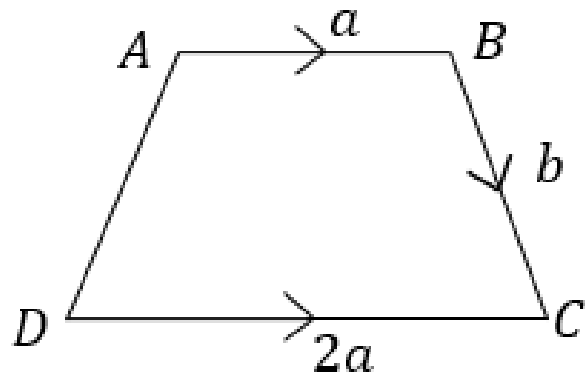
c  $\vec{OQ} = \vec{OP} + \vec{PQ} = \mathbf{p} + \mathbf{r}$

d  $\vec{PR} = \vec{PQ} + \vec{QR} = \mathbf{r} - \mathbf{p}$

Always write the journey you will take

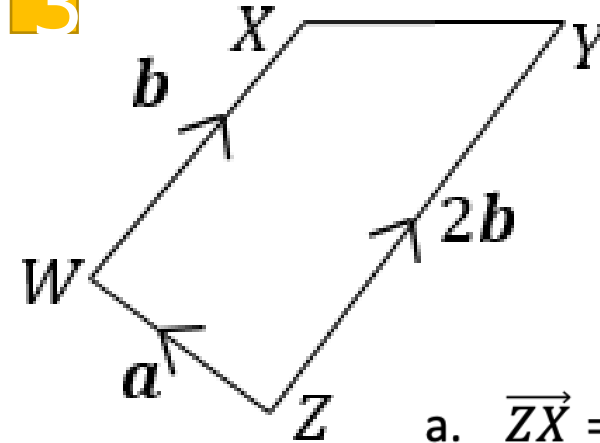
# Test your understanding

1



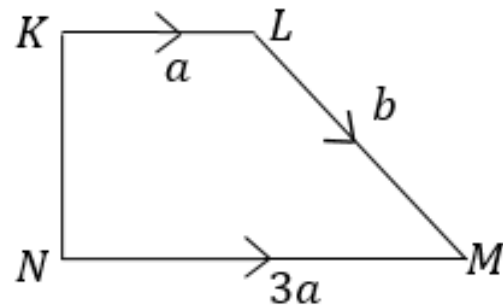
- a.  $\overrightarrow{BA} =$
- b.  $\overrightarrow{AC} =$
- c.  $\overrightarrow{DB} =$
- d.  $\overrightarrow{AD} =$

3



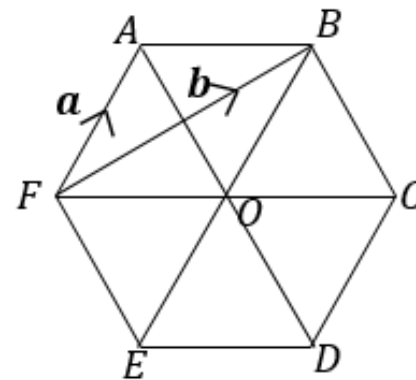
- a.  $\overrightarrow{ZX} =$
- b.  $\overrightarrow{YW} =$
- c.  $\overrightarrow{XY} =$
- d.  $\overrightarrow{XZ} =$

2



- a.  $\overrightarrow{MK} =$
- b.  $\overrightarrow{NL} =$
- c.  $\overrightarrow{NK} =$
- d.  $\overrightarrow{KN} =$

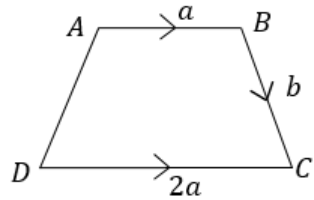
4



- a.  $\overrightarrow{AB} =$
- b.  $\overrightarrow{FO} =$
- c.  $\overrightarrow{AO} =$
- d.  $\overrightarrow{FD} =$

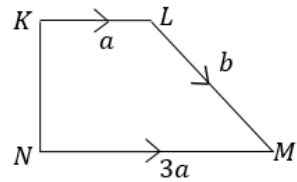
# Test your understanding

1



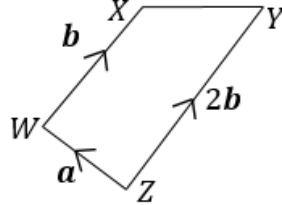
- $\overrightarrow{BA} =$
- $\overrightarrow{AC} =$
- $\overrightarrow{DB} =$
- $\overrightarrow{AD} =$

2



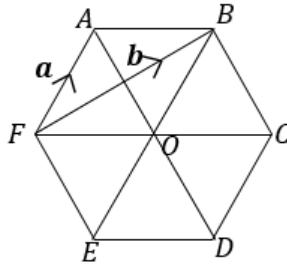
- $\overrightarrow{MK} =$
- $\overrightarrow{NL} =$
- $\overrightarrow{NK} =$
- $\overrightarrow{KN} =$

3



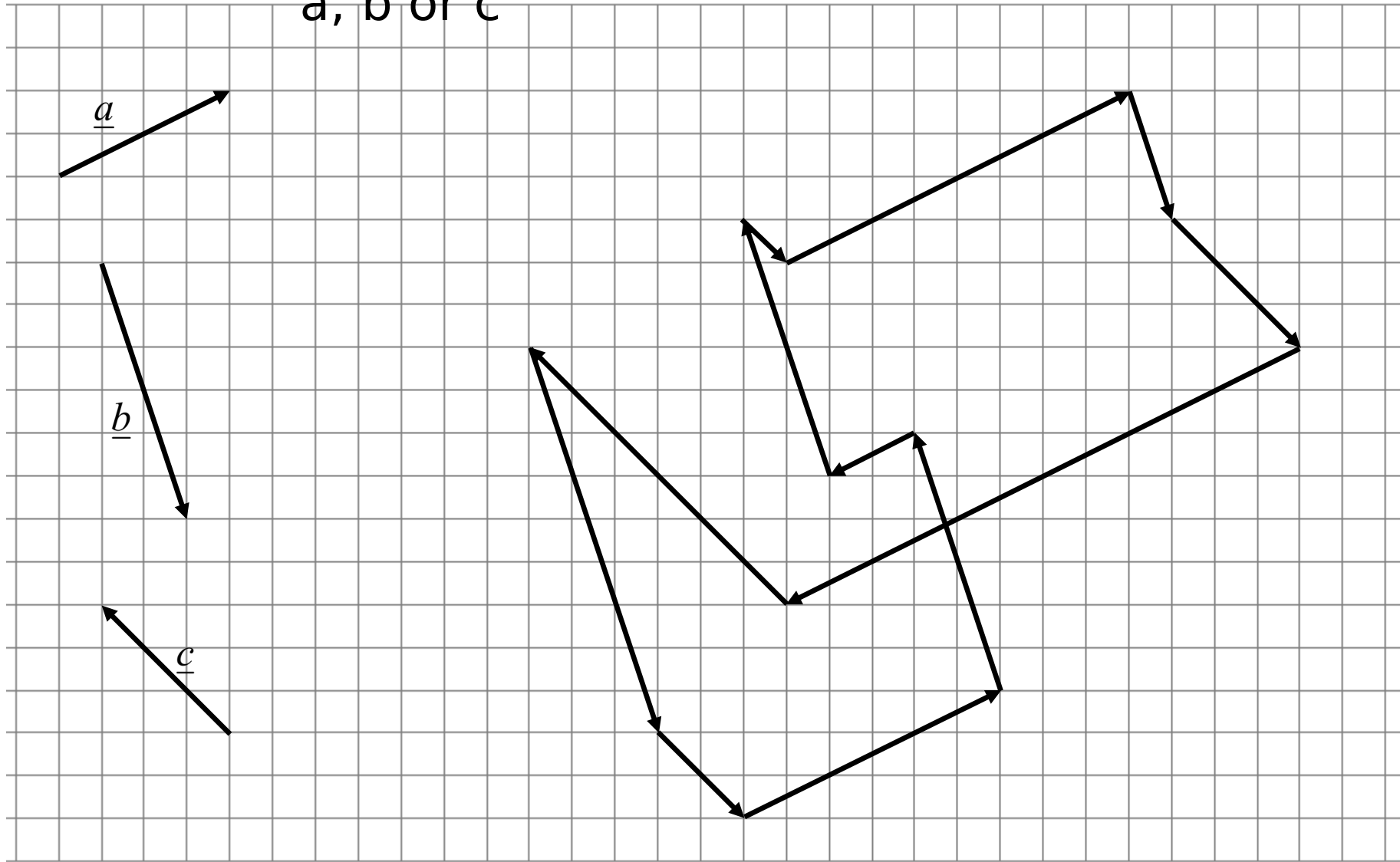
- $\overrightarrow{ZX} =$
- $\overrightarrow{YW} =$
- $\overrightarrow{XY} =$
- $\overrightarrow{XZ} =$

4



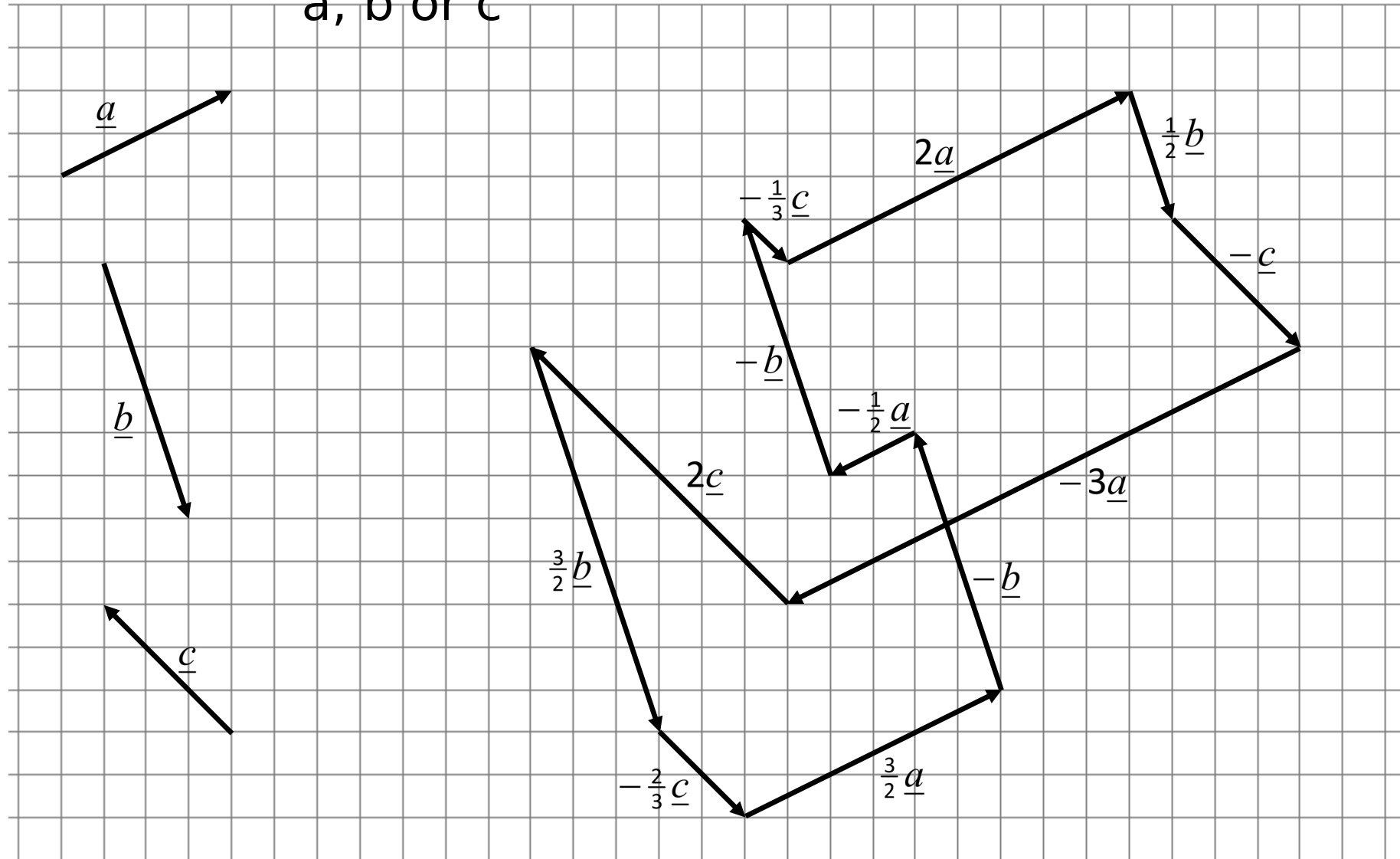
- $\overrightarrow{AB} =$
- $\overrightarrow{FO} =$
- $\overrightarrow{AO} =$
- $\overrightarrow{FD} =$

Label all vectors as multiples of  
a, b or c



Can you explain why it forms  
a loop?

Label all vectors as multiples of  
a, b or c



Can you explain why it forms  
a loop?

$$2\underline{a} - 3\underline{a} + \frac{3}{2}\underline{a} - \frac{1}{2}\underline{a} = 0$$

$$\frac{1}{2}\underline{b} + \frac{3}{2}\underline{b} - \underline{b} - \underline{b} = 0$$

$$-\underline{c} + 2\underline{c} - \frac{2}{3}\underline{c} - \frac{1}{3}\underline{c} = 0$$

# Vector Geometry - midpoints

Q  
2

P and Q are the midpoints of sides OA and OB.

$OP = \mathbf{p}$  and  $OQ = \mathbf{q}$

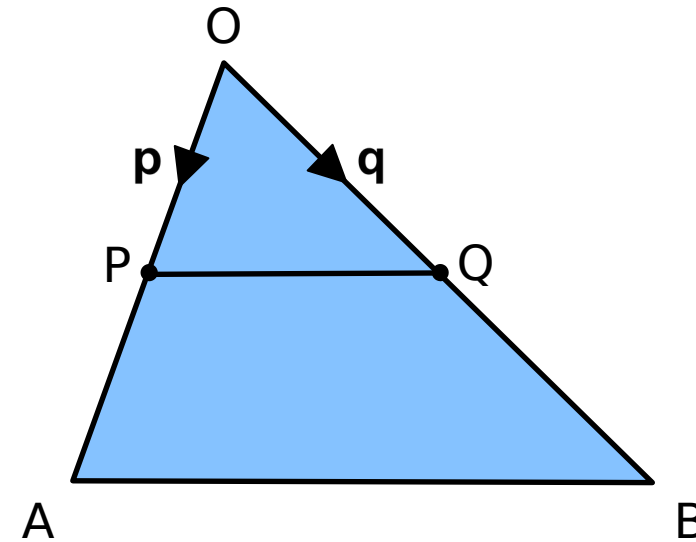
Find in terms of  $\mathbf{p}$  and  $\mathbf{q}$ .

**a**  $OA$

**b**  $BA$

**c**  $AQ$

**d**  $PB$



Always write the journey you will take

# Vector Geometry - midpoints

Q  
2

P and Q are the midpoints of sides OA and OB.

$\overrightarrow{OP} = \mathbf{p}$  and  $\overrightarrow{OQ} = \mathbf{q}$

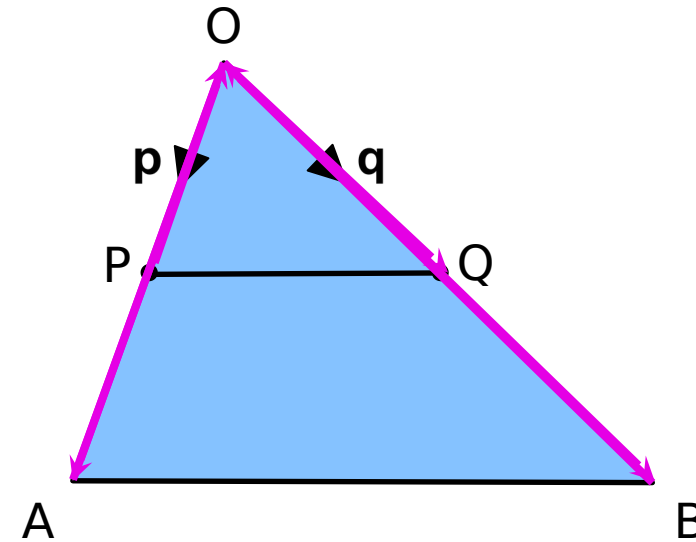
Find in terms of  $\mathbf{p}$  and  $\mathbf{q}$ .

**a**  $\overrightarrow{OA} = 2\mathbf{p}$

**b**  $\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA} = -2\mathbf{q} + 2\mathbf{p}$

**c**  $\overrightarrow{AQ} = \overrightarrow{AO} + \overrightarrow{OQ} = -2\mathbf{p} + \mathbf{q}$

**d**  $\overrightarrow{PB} = \overrightarrow{PO} + \overrightarrow{OB} = -\mathbf{p} + 2\mathbf{q}$



Always write the journey you will take

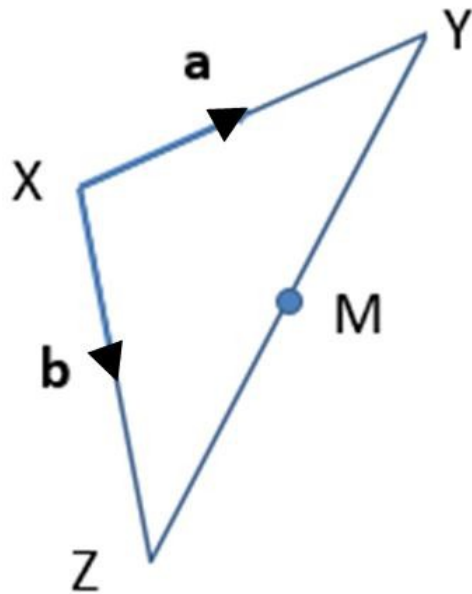
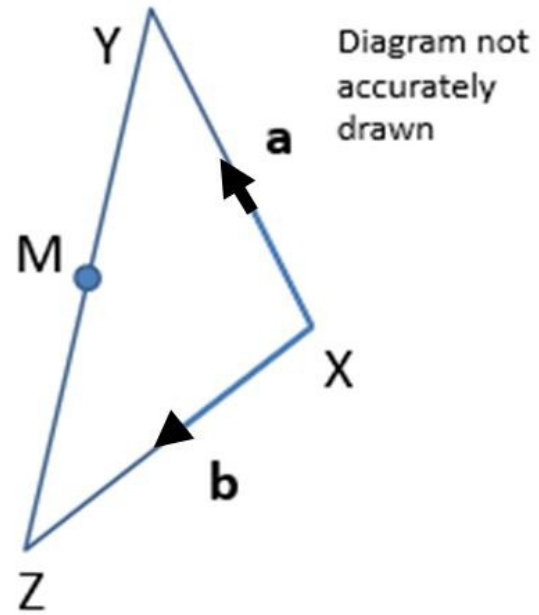


Diagram not  
accurately  
drawn

M is the midpoint of Y and Z

Find the vector  $\mathbf{YM}$





M is the midpoint of Y and Z

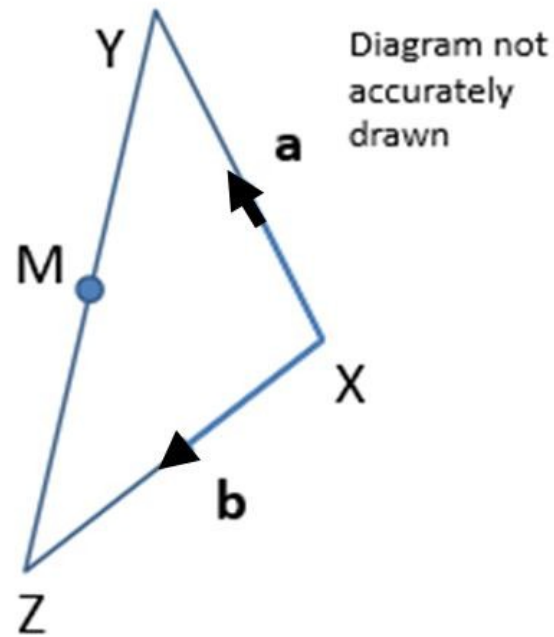
**Step 1:** Find the vector  $\overrightarrow{ZY}$

A  $-b + a$

B  $-a + b$

C  $a + b$

D  $-a - b$



M is the midpoint of Y and Z

$$\text{Vector } \overrightarrow{ZY} = -\mathbf{b} + \mathbf{a}$$

**Step 2:** Find vector  $\overrightarrow{ZM}$

A Both B and C are correct

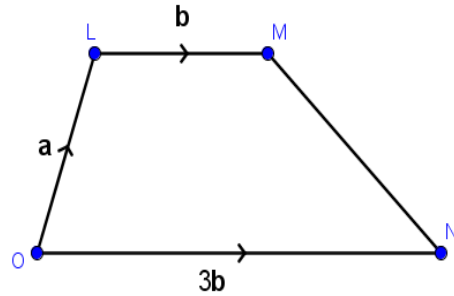
C  $\frac{1}{2}(\mathbf{a} - \mathbf{b})$

B  $\frac{1}{2}(\mathbf{b} - \mathbf{a})$

D  $\frac{1}{2}(\mathbf{a} + \mathbf{b})$

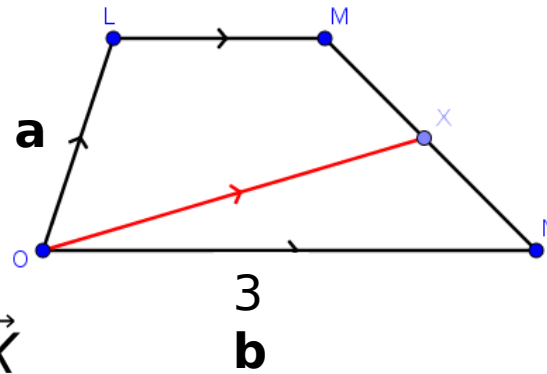
# TEST YOUR UNDERSTANDING

$X$  is the mid-point for the line  $MN$ . What is the position vector of  $X$ ? **b**



$$\overrightarrow{OX} = \overrightarrow{OL} + \overrightarrow{LM} + \overrightarrow{MX}$$

$$\text{but } \overrightarrow{MX} = \frac{1}{2} \overrightarrow{MN}$$



$$\begin{aligned} \overrightarrow{MN} &= -\mathbf{b} - \mathbf{a} + 3\mathbf{b} \\ &= -\mathbf{a} + 2\mathbf{b} \end{aligned}$$

$$\begin{aligned} \overrightarrow{OX} &= \mathbf{a} + \mathbf{b} + \frac{1}{2}(-\mathbf{a} + 2\mathbf{b}) \\ &= \frac{1}{2}\mathbf{a} + 2\mathbf{b} \end{aligned}$$

# vector match

Which routes do these describe?  
Match them to the vectors at the bottom.

$$c + e + m$$

$$m - \frac{b}{2}$$

$$\frac{b}{2}$$

$$\frac{c - b}{2}$$

$$b + c - \frac{b}{2}$$

$$\frac{c - e}{2}$$

$$b - c$$

$$c - 2m$$

$$b - 2m$$

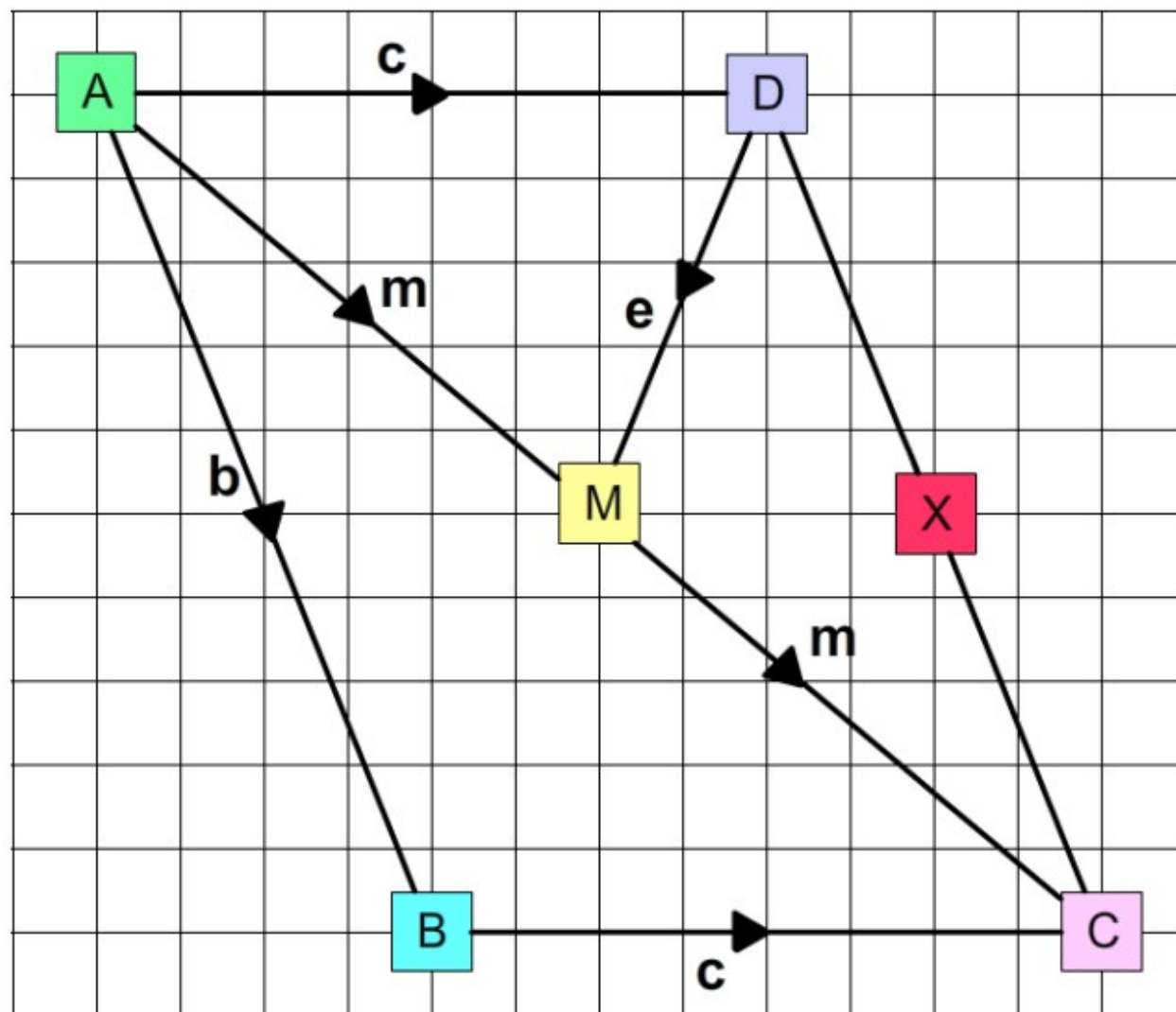
$$\frac{b + c}{2}$$

$$m - c - b$$

$$b + c - m - e$$

$$\frac{b - c}{2}$$

$$c + b - c$$



**Answers**

$\vec{BX}$

$\vec{MX}$

$\vec{DX}$

$\vec{DM}$

$\vec{BM}$

$\vec{DB}$

$\vec{AM}$

$\vec{AC}$

$\vec{AB}$

$\vec{AD}$

$\vec{AX}$

$\vec{CB}$

$\vec{MA}$

$\vec{BA}$

# Answers

$$c + e + m$$

$\overrightarrow{AC}$

$$\frac{c}{2} - e$$

$\overrightarrow{BX}$

$$m - c - b$$

$\overrightarrow{MA}$

$$m - \frac{b}{2}$$

$\overrightarrow{MX}$

$$b - c$$

$\overrightarrow{DB}$

$$b + c - m - e$$

$\overrightarrow{AD}$

$$\frac{b}{2}$$

$\overrightarrow{DX}$

$$c - 2m$$

$\overrightarrow{BA}$

$$\frac{b - c}{2}$$

$\overrightarrow{DM}$

$$\frac{c - b}{2}$$

$\overrightarrow{BM}$

$$b - 2m$$

$\overrightarrow{CB}$

$$c + b - c$$

$\overrightarrow{AB}$

$$b + c - \frac{b}{2}$$

$\overrightarrow{AX}$

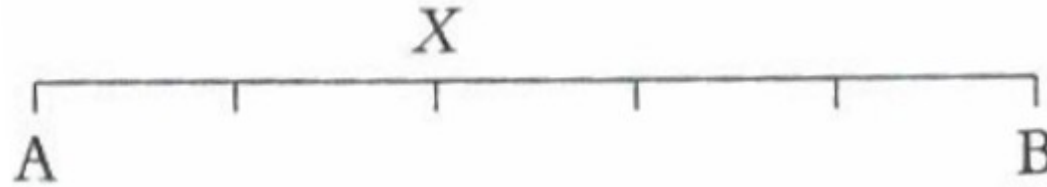
$$\frac{b + c}{2}$$

$\overrightarrow{AM}$

# Vector Geometry - ratios

Ratios can also be used in vector geometry

If  $X$  is a point on the line  $AB$  such that  $AX : XB = 2 : 3$ , label  $X$  on the line:

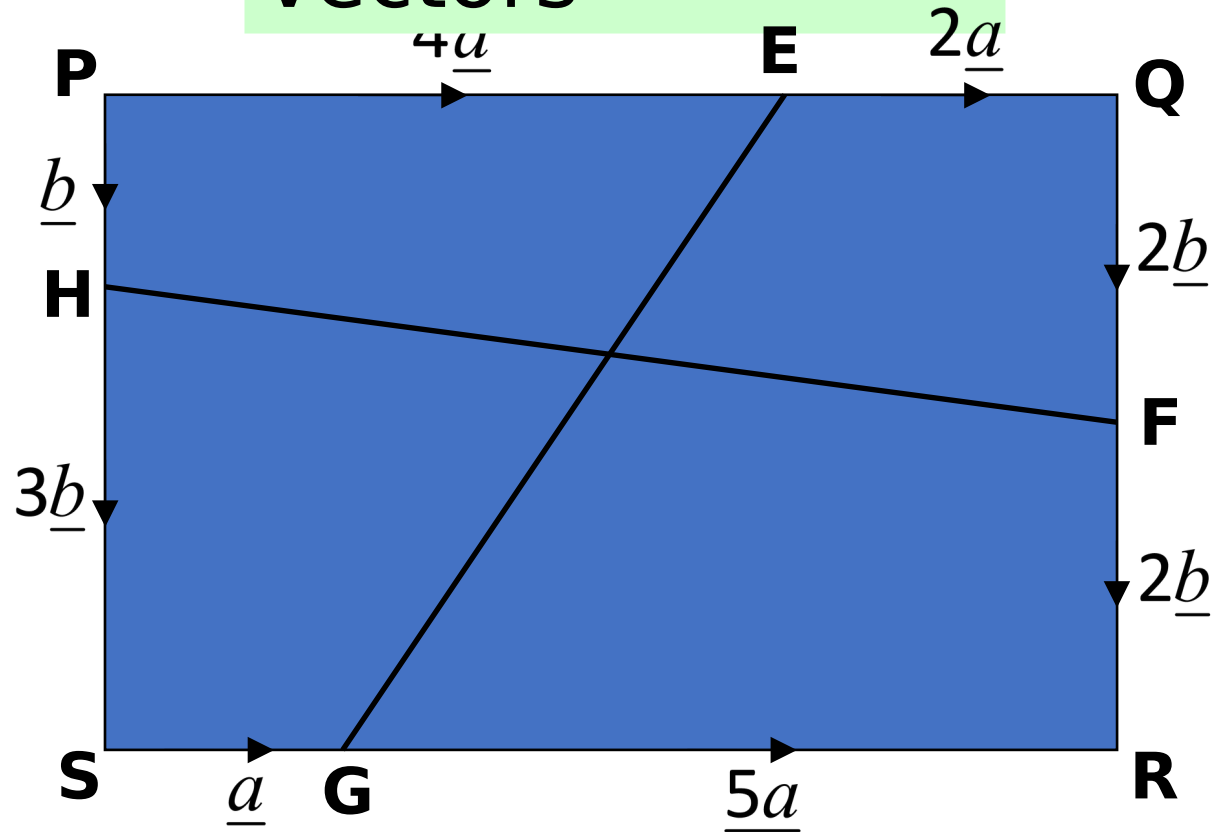


$$\vec{AX} = \boxed{?} \vec{AB}$$

$$\vec{XB} = \boxed{?} \vec{AB}.$$

What value have  
the hidden  
scalars?

# Comparing vectors



On your mini white boards, write down the **ratio of the lengths** of the lines:

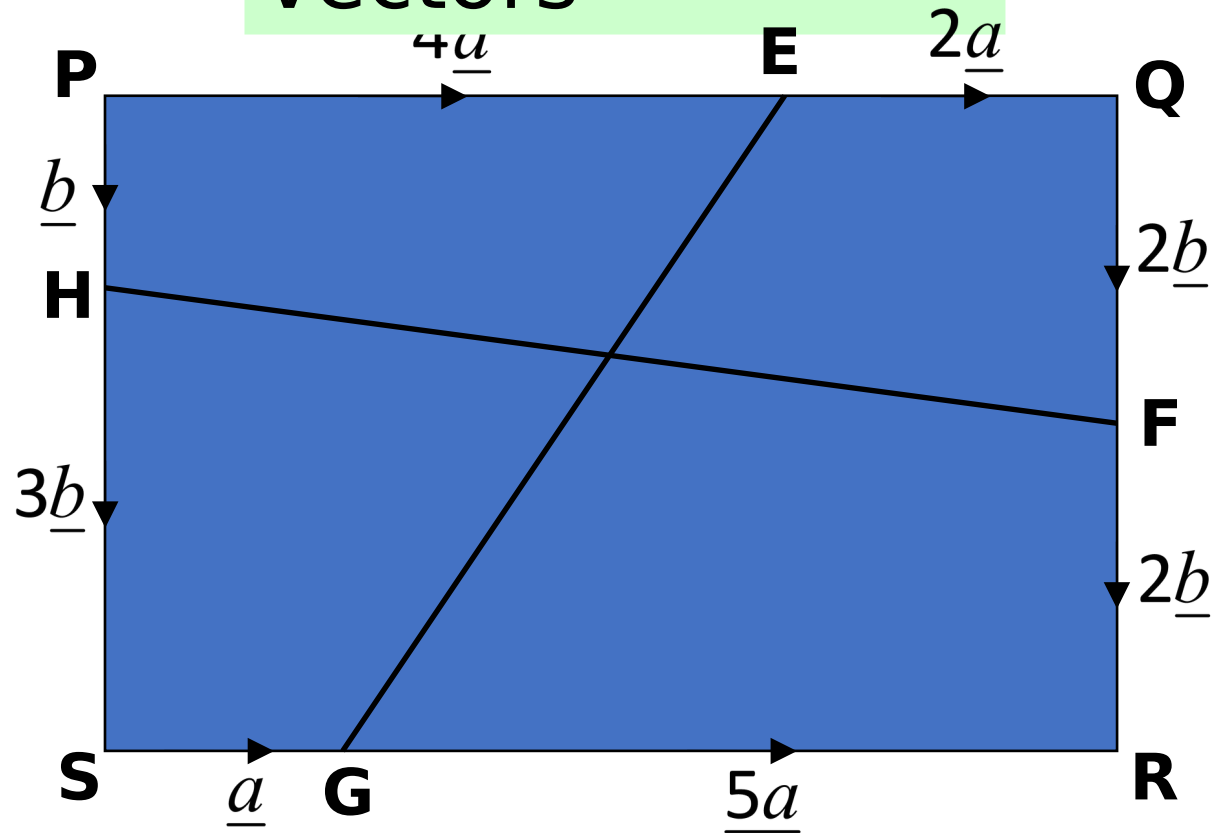
$$PE : EQ$$

$$SG : SR$$

$$PS : HS$$

$$QF : FR$$

# Comparing vectors



On your mini white boards, write down the **ratio of the lengths** of the lines:

$$PE : EQ \quad 2:1 \quad SG : SR \quad 1:5 \quad PS : HS \quad 4:3 \quad QF : FR \quad 1:1$$



# Vector Geometry - ratios

## EXAMPLE

$$AP : PB = 2 : 1$$

$\overrightarrow{OA} = \mathbf{a}$  and  $\overrightarrow{OB} = \mathbf{b}$   
Find in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

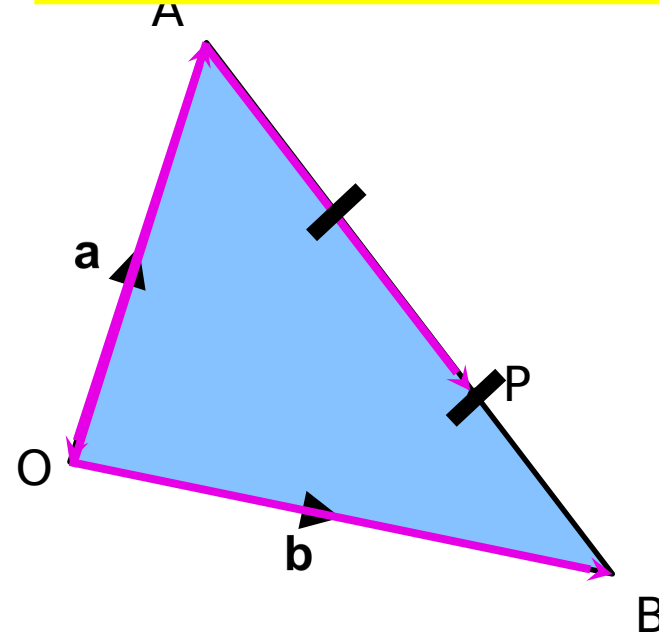
$$\mathbf{a} \quad \overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$

$$= -\mathbf{a} + \mathbf{b}$$

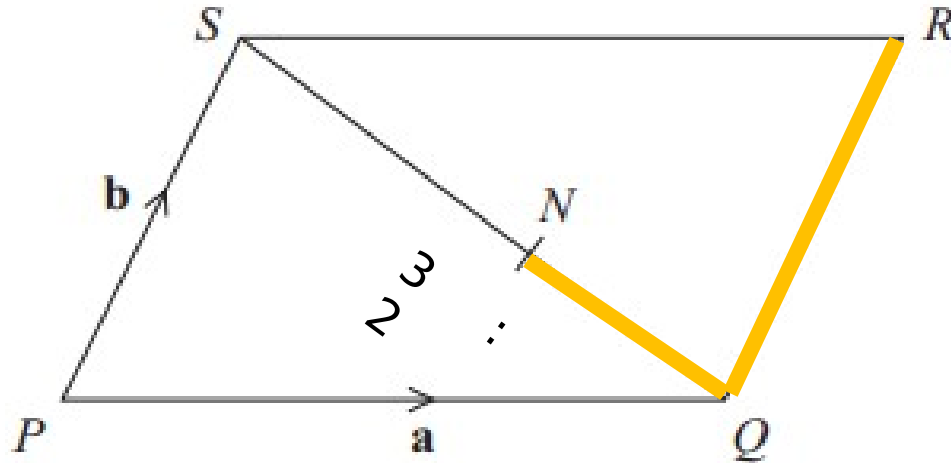
$$\mathbf{b} \quad \overrightarrow{AP} = \frac{2}{3} \overrightarrow{AB}$$
$$= \frac{2}{3}(-\mathbf{a} + \mathbf{b})$$

$$\mathbf{c} \quad \overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$$
$$= \mathbf{a} + \frac{2}{3}(-\mathbf{a} + \mathbf{b})$$
$$= \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$$

First label the diagram with the ratios!



# Vector Geometry - ratios



$PQRS$  is a parallelogram.

$N$  is the point on  $SQ$  such that  $SN : NQ = 3 : 2$

$$\vec{PQ} = \mathbf{a} \quad \vec{PS} = \mathbf{b}$$

(a) Write down, in terms of  $\mathbf{a}$  and  $\mathbf{b}$ , an expression for  $\vec{SQ}$ .

(b) Express  $\vec{NR}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

a  $\vec{SQ} = -\mathbf{b} + \mathbf{a}$

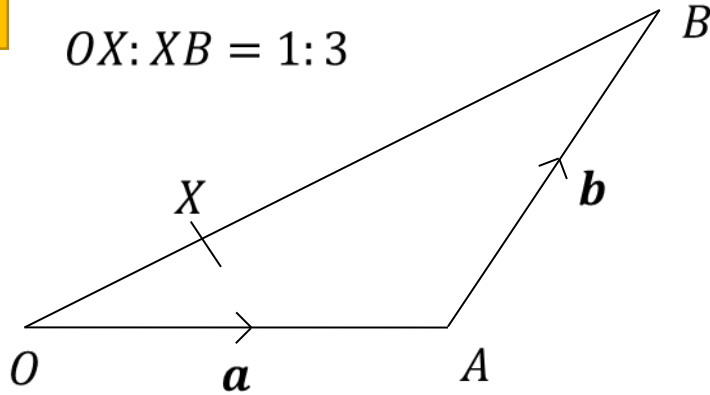
b

?

# Test your understanding

A

$$OX:XB = 1:3$$



$$\overrightarrow{AX} = \text{First Step?}$$

=

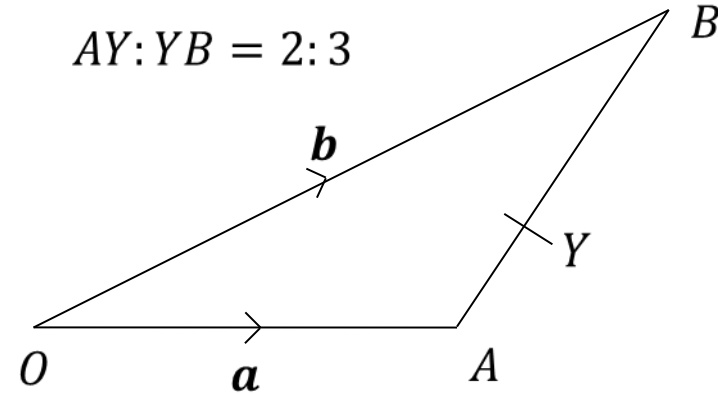
=

=

?

B

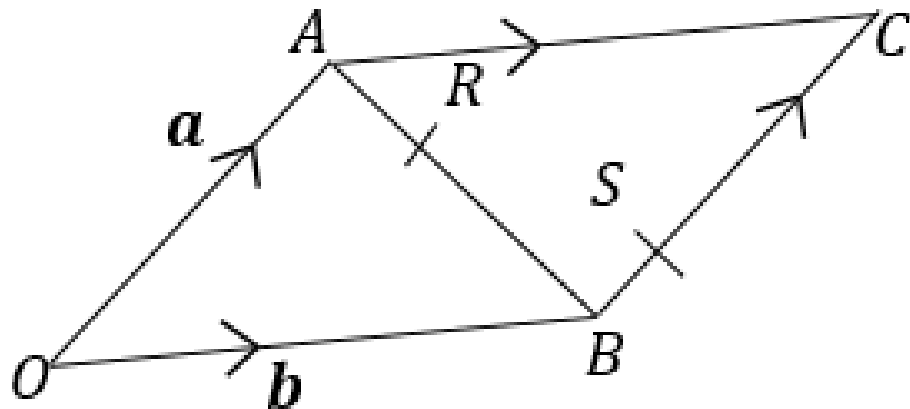
$$AY:YB = 2:3$$



$$\overrightarrow{OY} = \text{First Step?}$$

?

$OACB$  is a parallelogram.  $R$  is a point such that  $AR:RB = 2:3$ .  
 $S$  is a point such that  $BS:SC = 1:3$ .



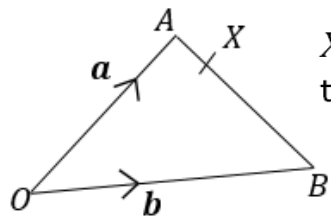
a.  $\vec{OR} =$

b.  $\vec{BS} =$

c.  $\vec{OS} =$

d.  $\vec{RS} =$

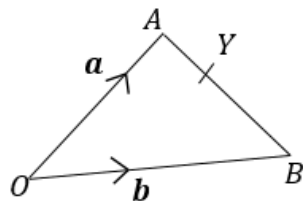
1



$X$  is a point such  
that  $AX:XB = 1:4$

- $\overrightarrow{AB} =$
- $\overrightarrow{AX} =$
- $\overrightarrow{OX} =$
- $\overrightarrow{BX} =$

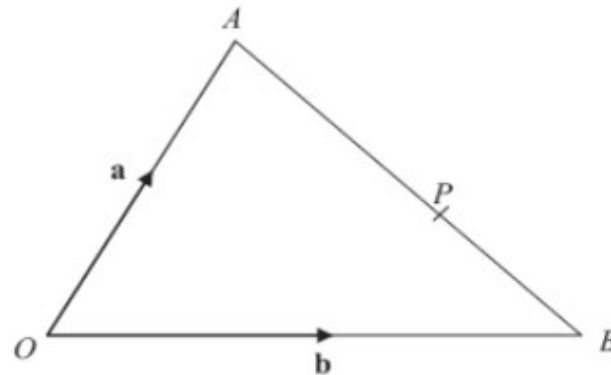
2



$Y$  is a point such  
that  $YB = 2AY$

- $\overrightarrow{AY} =$
- $\overrightarrow{OY} =$
- $\overrightarrow{YO} =$

3



- Find  $\overrightarrow{AB}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

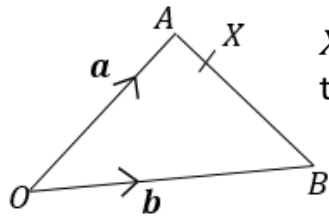
$$\overrightarrow{AB} = -\mathbf{a} + \mathbf{b} \text{ or } \mathbf{b} - \mathbf{a}$$

- $P$  is on  $AB$  such that  $AP:PB = 3:2$ .

$$\text{Show that } \overrightarrow{OP} = \frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$$

# Quick practice

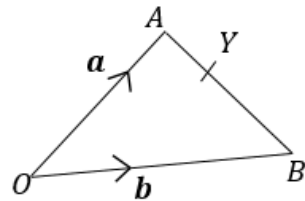
1



$X$  is a point such that  $AX:XB = 1:4$

- $\overrightarrow{AB} =$
- $\overrightarrow{AX} =$
- $\overrightarrow{OX} =$
- $\overrightarrow{BX} =$

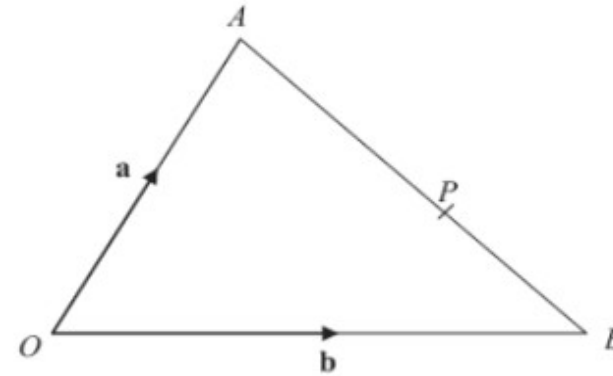
2



$Y$  is a point such that  $YB = 2AY$

- $\overrightarrow{AY} =$
- $\overrightarrow{OY} =$
- $\overrightarrow{YO} =$

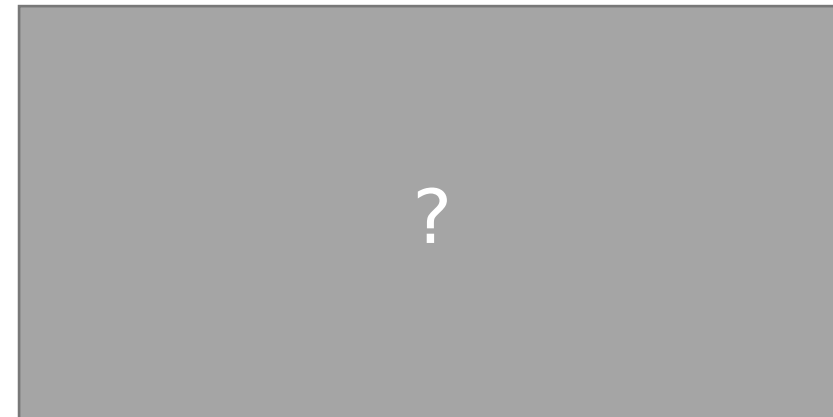
3



a) Find  $\overrightarrow{AB}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

b)  $P$  is on  $AB$  such that  $AP:PB = 3:2$ .

Show that  $\overrightarrow{OP} = \frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$



# Exam style question

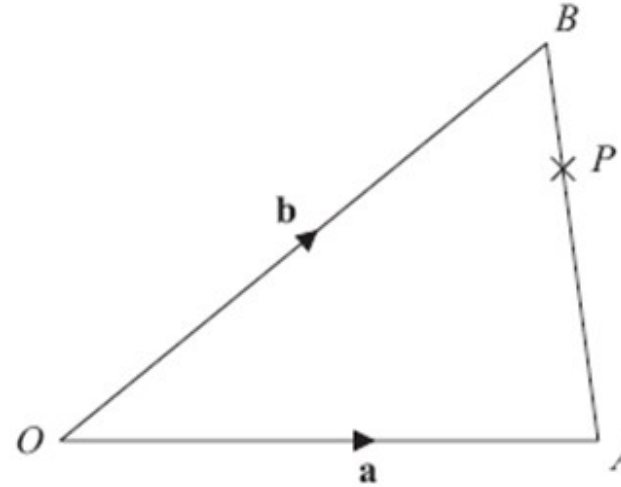


Diagram **NOT**  
accurately drawn

$OAB$  is a triangle.

$$\vec{OA} = \mathbf{a}$$

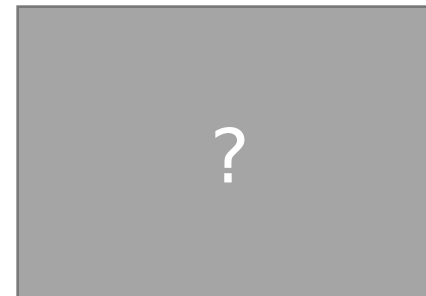
$$\vec{OB} = \mathbf{b}$$

(a) Find  $\vec{AB}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .



$P$  is the point on  $AB$  such that  $AP : PB = 3 : 1$

(b) Find  $\vec{OP}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .  
Give your answer in its simplest form.



You **MUST**  
expand  
and  
simplify.

# All together

Q  
5

OPQR is a trapezium.

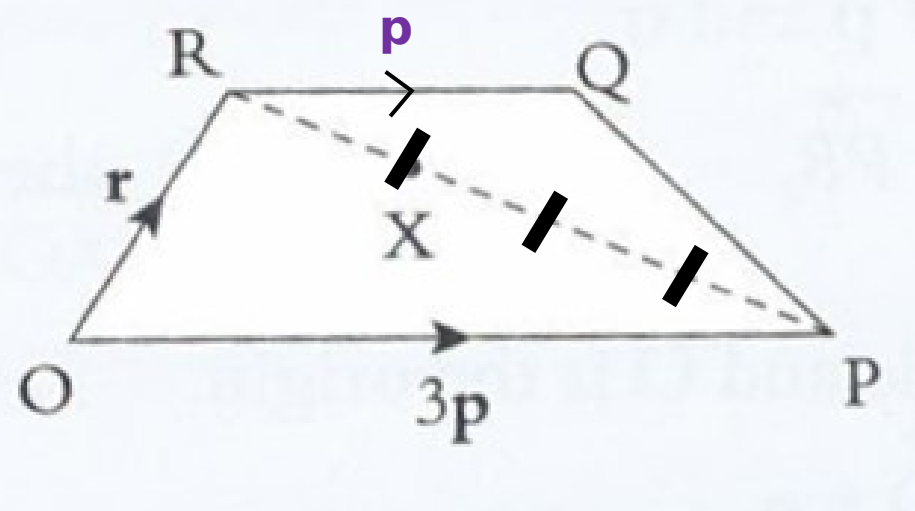
$$\overrightarrow{OP} = 3\mathbf{p} \text{ and } \overrightarrow{OR} = \mathbf{r}$$

$$\overrightarrow{RQ} = \frac{1}{3} \overrightarrow{OP}$$

X lies on RP such that  $RX : XP = 1 : 3$

➔ **NOTE:**  $RX : XP = 1 : 3$  means that  $\overrightarrow{RX} = \frac{1}{4} \overrightarrow{RP}$ .

First label the diagram with the ratios!



a Find in terms of  $\mathbf{p}$  and  $\mathbf{r}$

- i  $\overrightarrow{OQ}$ ,      ii  $\overrightarrow{RP}$ ,      iii  $\overrightarrow{RX}$ ,      iv  $\overrightarrow{OX}$ .

b What do your answers for  $\overrightarrow{OQ}$  and  $\overrightarrow{OX}$  tell you about the points O, X and Q?



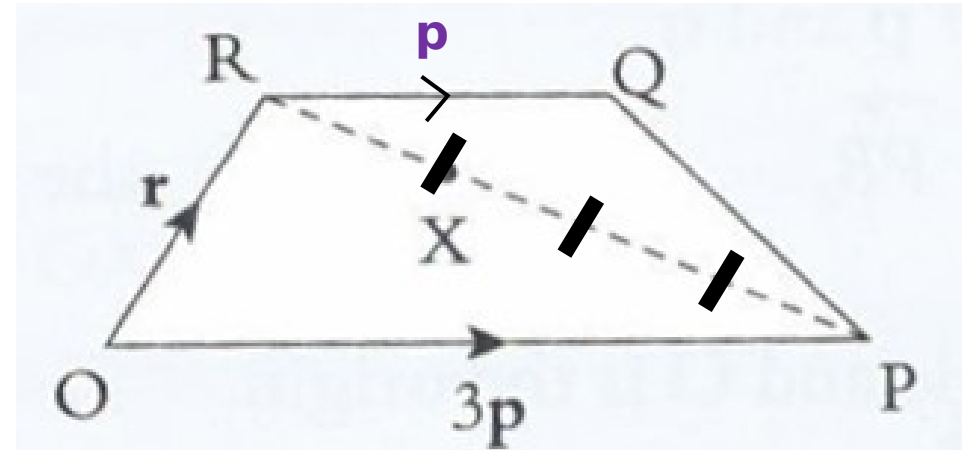
# SOLUTION

$$\text{a i } \vec{OQ} = \vec{OR} + \vec{RQ} \\ = \mathbf{r} + \mathbf{p}$$

$$\text{ii } \vec{RP} = \vec{RO} + \vec{OP} \\ = -\mathbf{r} + 3\mathbf{p}$$

$$\text{iii } \vec{RX} = \frac{1}{4} \vec{RP} \\ = \frac{1}{4}(-\mathbf{r} + 3\mathbf{p}) \\ = \frac{3}{4}\mathbf{p} - \frac{1}{4}\mathbf{r}$$

$$\text{iv } \vec{OX} = \vec{OR} + \vec{RX} = \mathbf{r} + \frac{3}{4}\mathbf{p} - \frac{1}{4}\mathbf{r} \\ = \frac{3}{4}\mathbf{r} + \frac{3}{4}\mathbf{p} \\ = \frac{3}{4}(\mathbf{r} + \mathbf{p})$$



$$\text{b } \vec{OQ} = \mathbf{r} + \mathbf{p} \text{ and } \vec{OX} = \frac{3}{4}(\mathbf{r} + \mathbf{p}) \\ \text{So } \vec{OX} = \frac{3}{4} \vec{OQ}$$

This means that OX and OQ are **parallel**.  
The points O lies on both of these line segments.  
So the points O, X and Q are **collinear**.

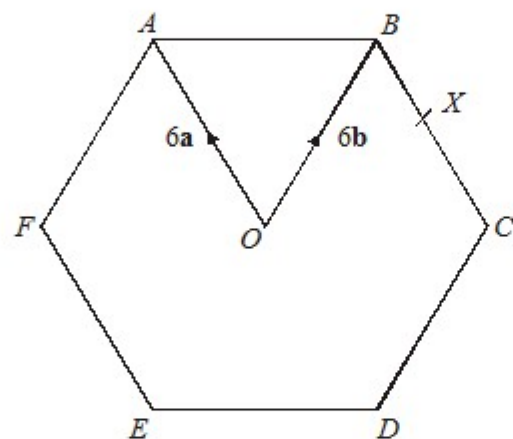


Diagram NOT  
accurately drawn

The diagram shows a regular hexagon  $ABCDEF$  with centre  $O$ .

$$\vec{OA} = 6\mathbf{a} \quad \vec{OB} = 6\mathbf{b}$$

(a) Express in terms of  $\mathbf{a}$  and/or  $\mathbf{b}$

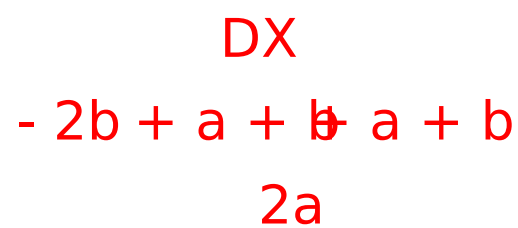
(i)  $\vec{AB}$ ,

.....

(ii)  $\vec{EF}$ .

$X$  is the midpoint of  $BC$ .

(b) Express  $\vec{EX}$  in terms of  $\mathbf{a}$  and/or  $\mathbf{b}$

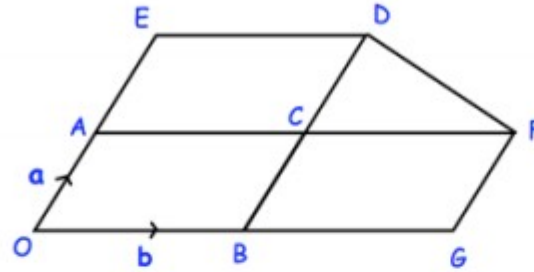


$ABCDEF$  is a regular hexagon.

(a) Find the vector  $\overrightarrow{AC}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ . a + b

(b) Prove that  $AB$  is parallel to  $DX$ .

1. In the diagram OBDE and OAFG are parallelograms.  
 B is the midpoint of OG.  
 A is the midpoint of OE.  
 $\overrightarrow{OA} = \mathbf{a}$  and  $\overrightarrow{OB} = \mathbf{b}$



- (a) Express, in terms of  $\mathbf{a}$  and  $\mathbf{b}$ , the following vectors.  
 Give your answers in their simplest form.

(i)  $\overrightarrow{OC}$

$$\underline{\underline{\mathbf{a} + \mathbf{b}}}$$

(1)

(ii)  $\overrightarrow{BA}$

$$\underline{\underline{\mathbf{a} - \mathbf{b}}}$$

(1)

(iii)  $\overrightarrow{DF}$

$$\underline{\underline{\mathbf{b} - \mathbf{a}}}$$

(1)

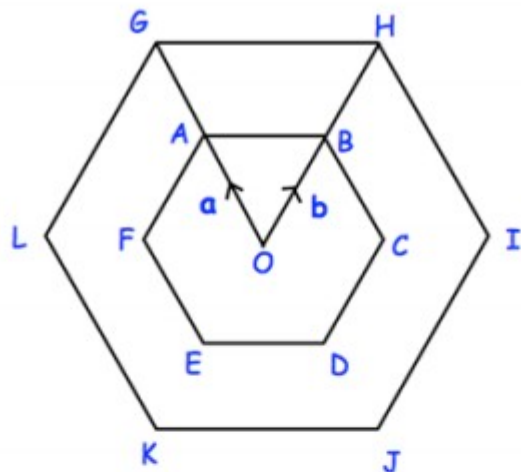
- (b) Show  $\overrightarrow{EG}$  and  $\overrightarrow{DF}$  are parallel.

$$\begin{aligned}\overrightarrow{EG} &= 2\mathbf{b} - 2\mathbf{a} \\ \overrightarrow{DF} &= \mathbf{b} - \mathbf{a}\end{aligned}$$

$$\begin{aligned}\overrightarrow{EG} &= 2\overrightarrow{DF} \\ \therefore \text{they are parallel}\end{aligned}$$

(2)

2.



ABCDEF and GHIJKL are regular hexagons with centre O. GHIJKL is an enlargement of ABCDEF, with scale factor 2.

$\overrightarrow{OA} = \mathbf{a}$  and  $\overrightarrow{OB} = \mathbf{b}$

(a) Write the vector  $\overrightarrow{AB}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\underline{\underline{\mathbf{b} - \mathbf{a}}}$$

(1)

(b) Write the vector  $\overrightarrow{OG}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\underline{\underline{2\mathbf{a}}}$$

(1)

(c) Write the vector  $\overrightarrow{OE}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\underline{\underline{-\mathbf{b}}}$$

(1)

(d) Write the vector  $\overrightarrow{FC}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\underline{\underline{2\mathbf{b} - 2\mathbf{a}}}$$

(1)

(e) Write the vector  $\overrightarrow{IK}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\underline{\underline{2\mathbf{a} - 4\mathbf{b}}}$$

(1)

(f) Write the vector  $\overrightarrow{LI}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\underline{\underline{4\mathbf{b} - 4\mathbf{a}}}$$

(1)

(g) Write the vector  $\overrightarrow{LG}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\underline{\underline{2\mathbf{b}}}$$

(1)

(h) Write the vector  $\overrightarrow{JG}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\underline{\underline{4\mathbf{a}}}$$

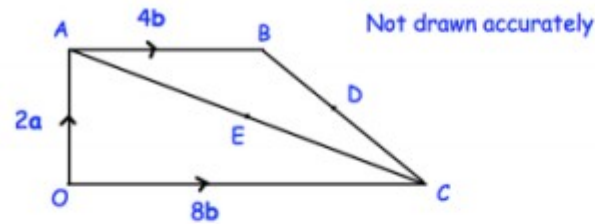
(1)

(i) Write the vector  $\overrightarrow{DL}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\underline{\underline{3\mathbf{a} - 2\mathbf{b}}}$$

(1)

3. OABC is a trapezium.  
 Point D is the midpoint of BC.  
 Point E is the midpoint of AC.  
 $\overrightarrow{OA} = 2\mathbf{a}$   $\overrightarrow{AB} = 4\mathbf{b}$  and  $\overrightarrow{OC} = 8\mathbf{b}$



- (a) Write these vectors in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

(i)  $\overrightarrow{OB}$

$$\underline{2\mathbf{a} + 4\mathbf{b}}$$

(1)

(ii)  $\overrightarrow{AC}$

$$\underline{8\mathbf{b} - 2\mathbf{a}}$$

(1)

(iii)  $\overrightarrow{AE}$

$$\underline{4\mathbf{b} - \mathbf{a}}$$

(1)

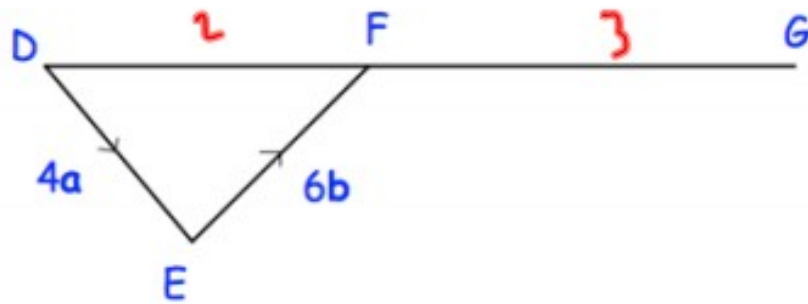
- (b) Show  $\overrightarrow{ED}$  and  $\overrightarrow{OC}$  are parallel.

$$\begin{aligned} \overrightarrow{OC} &= 8\mathbf{b} \\ \overrightarrow{ED} &= \underline{\mathbf{a}} - 4\mathbf{b} + 4\mathbf{b} + \frac{1}{2}(-4\mathbf{b} - 2\mathbf{a} + 8\mathbf{b}) \\ \overrightarrow{ED} &= \underline{\mathbf{a}} - 4\mathbf{b} + 4\mathbf{b} - 2\mathbf{b} - \underline{\mathbf{a}} + 4\mathbf{b} \\ \overrightarrow{ED} &= 2\mathbf{b} \quad \overrightarrow{OC} = 4\overrightarrow{ED} \\ &\therefore \text{parallel} \end{aligned}$$

(3)

4. DFG is a straight line.

$\overrightarrow{DE} = 4\mathbf{a}$  and  $\overrightarrow{EF} = 6\mathbf{b}$



- (a) Write down the vector  $\overrightarrow{DF}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$

$$\underline{4\mathbf{a} + 6\mathbf{b}}$$

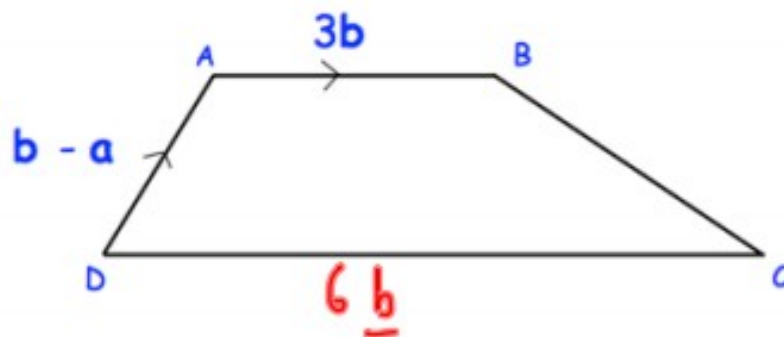
(1)

- (b)  $DF : FG = 2:3$

Work out the vector  $\overrightarrow{DG}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$   
Give your answer in its simplest form.

$$\begin{aligned} & (2\mathbf{a} + 3\mathbf{b}) \times 5 \\ & 10\mathbf{a} + 15\mathbf{b} \end{aligned}$$

5. ABCD is a trapezium



AB and DC are parallel.  
 $DC = 2AB$

- (a) Write down the vector  $\overrightarrow{DC}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$

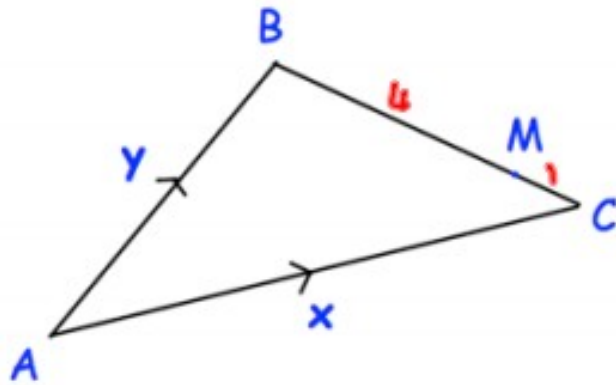
$$\underline{6b} \quad (1)$$

- (b) Work out the vector  $\overrightarrow{BC}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$   
Give your answer in its simplest form.

$$\underline{-3b - b + a + 6b}$$

$$\underline{2b + a} \quad (2)$$





ABC is a triangle.

M lies on BC such that  $BM = \frac{4}{5} BC$

Express these vectors in terms of  $\mathbf{x}$  and  $\mathbf{y}$

(a)  $\overrightarrow{BC}$

$$\underline{-y + x} \quad (1)$$

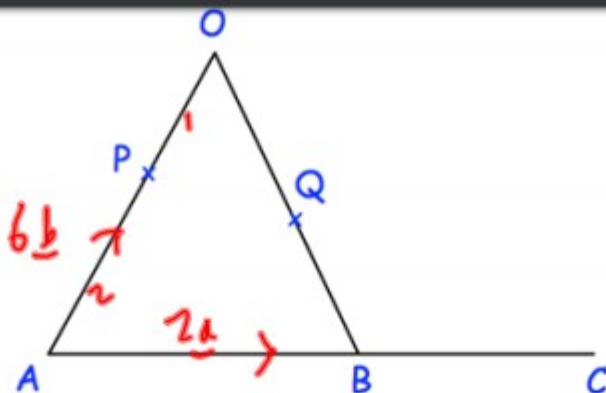
(b)  $\overrightarrow{BM}$

$$\underline{-\frac{4}{5}y + \frac{4}{5}x} \quad (1)$$

(c)  $\overrightarrow{AM}$

$$\underline{y - \frac{4}{5}y + \frac{4}{5}x}$$

$$\underline{\frac{1}{5}y + \frac{4}{5}x} \quad (1)$$



AOB is a triangle.  
P is a point on AO.

$$\overrightarrow{AB} = 2\mathbf{a}$$

$$\overrightarrow{AO} = 6\mathbf{b}$$

$$AP:PO = 2:1$$

(a) Find the vector  $\overrightarrow{OB}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$

$$\underline{2\mathbf{a} - 6\mathbf{b}}$$

.....  
(1)

Q is the midpoint of OB.  
B is the midpoint of AC.

(b) Show PQC is a straight line.

$$\overrightarrow{PQ} = 2\mathbf{b} + \mathbf{a} - 3\mathbf{b}$$

$$\overrightarrow{PQ} = \mathbf{a} - \mathbf{b}$$

$$\overrightarrow{QC} = 3\overrightarrow{PQ}$$

$$\overrightarrow{QC} = \mathbf{a} - 3\mathbf{b} + 2\mathbf{a}$$

$$\overrightarrow{QC} = 3\mathbf{a} - 3\mathbf{b}$$

QC and PQ are parallel and also both pass through the point Q, therefore PQC must be a straight line. (co-linear)

(3)

4.

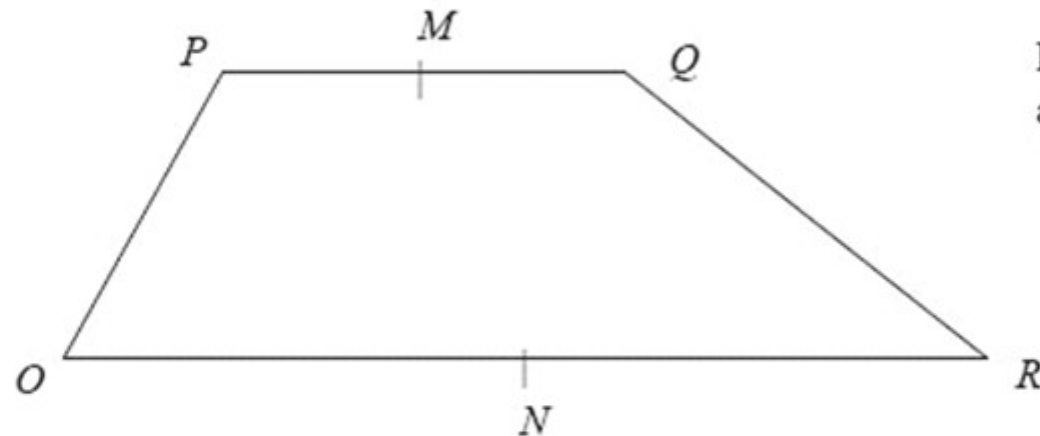


Diagram **NOT**  
accurately drawn

$OPQR$  is a trapezium with  $PQ$  parallel to  $OR$ .

$$\overrightarrow{OP} = 2\mathbf{b} \quad \overrightarrow{PQ} = 2\mathbf{a} \quad \overrightarrow{OR} = 6\mathbf{a}$$

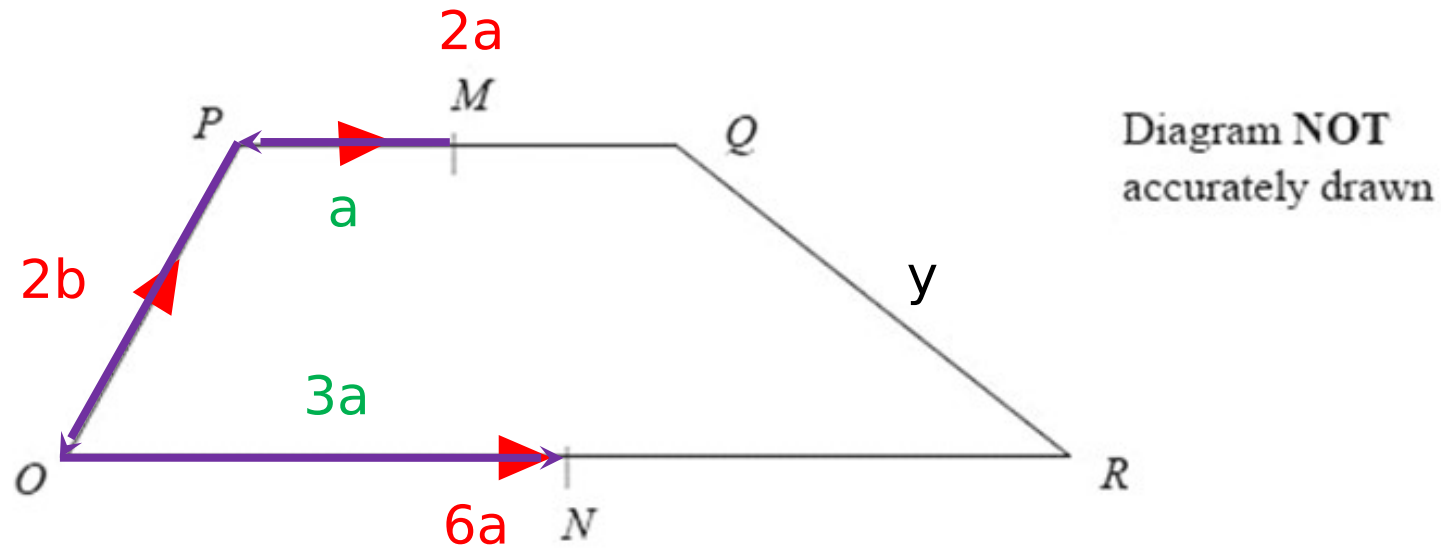
$M$  is the midpoint of  $PQ$  and  $N$  is the midpoint of  $OR$ .

(a) Find the vector  $\overrightarrow{MN}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$X$  is the midpoint of  $MN$  and  $Y$  is the midpoint of  $QR$ .

(b) Prove that  $XY$  is parallel to  $OR$ .

4.



$OPQR$  is a trapezium with  $PQ$  parallel to  $OR$ .

$$\overrightarrow{OP} = 2\mathbf{b} \quad \overrightarrow{PQ} = 2\mathbf{a} \quad \overrightarrow{OR} = 6\mathbf{a}$$

$$-\mathbf{a} - 2\mathbf{b} + 3\mathbf{a}$$

$$2\mathbf{a} - 2\mathbf{b}$$

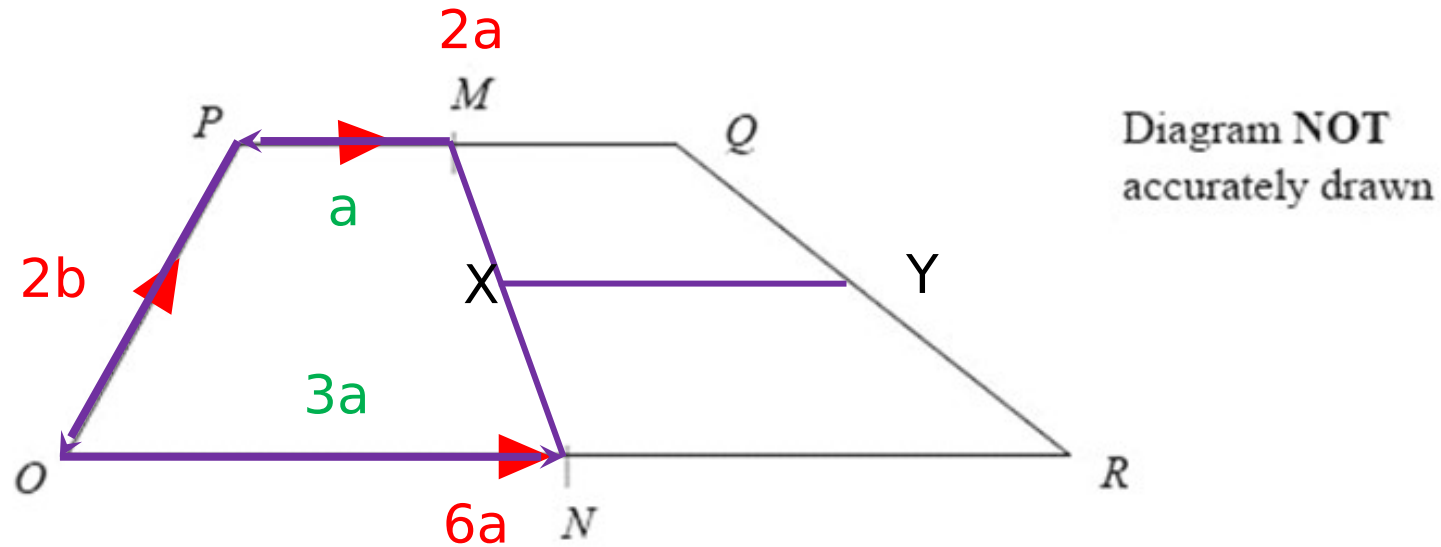
$M$  is the midpoint of  $PQ$  and  $N$  is the midpoint of  $OR$ .

(a) Find the vector  $\overrightarrow{MN}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$X$  is the midpoint of  $MN$  and  $Y$  is the midpoint of  $QR$ .

(b) Prove that  $XY$  is parallel to  $OR$ .

4.



$OPQR$  is a trapezium with  $PQ$  parallel to  $OR$ .

$$\overrightarrow{OP} = 2\mathbf{b} \quad \overrightarrow{PQ} = 2\mathbf{a} \quad \overrightarrow{OR} = 6\mathbf{a}$$

$M$  is the midpoint of  $PQ$  and  $N$  is the midpoint of  $OR$ .

$$2\mathbf{a} - 2\mathbf{b}$$

(a) Find the vector  $\overrightarrow{MN}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$X$  is the midpoint of  $MN$  and  $Y$  is the midpoint of  $QR$ .

(b) Prove that  $XY$  is parallel to  $OR$ .

1<sup>st</sup> Work out  $QR$

$$-2\mathbf{a} - 2\mathbf{b} + 6\mathbf{a}$$

$$4\mathbf{a} - 2\mathbf{b}$$

$QY$  will be half this

$$2\mathbf{a} - \mathbf{b}$$

Find  $XY$

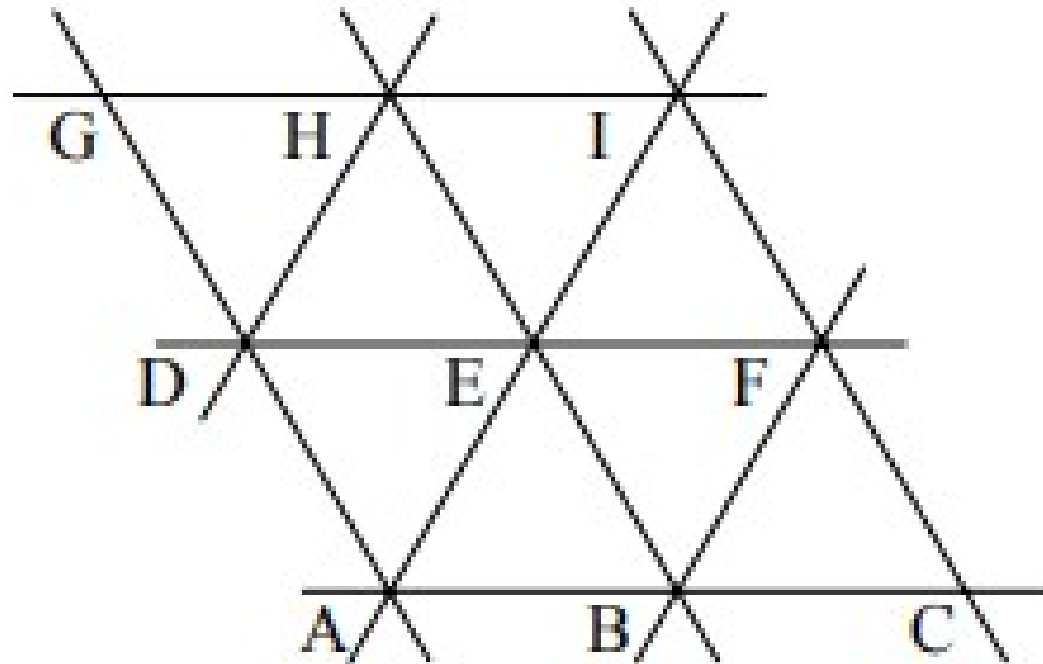
$$XM = \frac{1}{2} (2\mathbf{a} - 2\mathbf{b})$$

$$XM = -\mathbf{a} + \mathbf{b}$$

$$XY = -\mathbf{a} + \mathbf{b} + \mathbf{a} + 2\mathbf{a} -$$

$$XY = 2\mathbf{a}$$

$$AC = \underline{s} \text{ and } \overrightarrow{AD} = \overrightarrow{t}$$



What could the question be?